Exam. Code : 103205
Subject Code : 1203

B.A./B.Sc. 5th Semester

MATHEMATICS

Paper—II

(Number Theory)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **five** questions in all selecting at least two questions from each Section.

SECTION—A

I. (a) If $4x-y$ is a multiple of 3, show that $4x^2 + 7xy - 2y^2$ is divisible by 9. 5

(b) Show that the product of m consecutive integers is divisible by $\underline{|m}$. 5

II. (a) Show that if x and y are odd integers, then $16 \mid (x^4 + y^4 - 2)$. 5

(b) If a, b are any two integers, not both zero, and m is a positive integer, prove that gcd (ma, mb) = m·gcd (a, b). 5

III. (a) Find all integers x, y such that $147x + 258y = 369$. 5

(b) For any prime $p > 3$, prove that $p^2 - 1$ is divisible by 24. 5

IV. (a) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, prove that $a + c \equiv (b + d) \bmod m$ and $ac \equiv bd \pmod{m}$. 5

(b) Show that any positive integer of the form 3K +2 has a prime factor of the form 3K + 2. 5

V. (a) Show that for every prime p > 5, either $p^2 - 1$ or p^2+1 is divisible by 10. 5

(b) Solve $140x \equiv 133 \pmod{301}$. 5

SECTION—B

VI. (a) Find the least positive integer which when divided by 5, 6, 7 gives remainder 3, 1, 4 respectively. 5

(b) If g.c.d.(a, 133) = g.c.d.(b, 133) = 1, prove that $a^{18} \equiv b^{18} \pmod{133}$. 5

VII. (a) Show that 19 is a prime using converse of Wilson's theorem. 5

(b) Find remainder when $\lfloor 15$ is divided by 17. 5

VIII.(a) For even integer n, prove that $\phi(2n) = 2\phi(n)$ where $\phi(n)$ is Euler's phi-function. 5

(b) If n+2, n both are primes, then show that $\phi(n+2) = \phi(n)+2$. 5

IX. (a) Show that $a^{560} \equiv 1 \pmod{561}$ if g.c.d. (a, 561) =1, however 561 is not a prime. 5

(b) Find n such that $\phi(n) = 97$. 5

X. (a) If x and y are real numbers, prove that :

$[x] + [y] \leq [x+y]$. 5

(b) Find a positive integer n such that :

$\mu(n) + \mu(n+1) + \mu(n+2) = 3$. 5